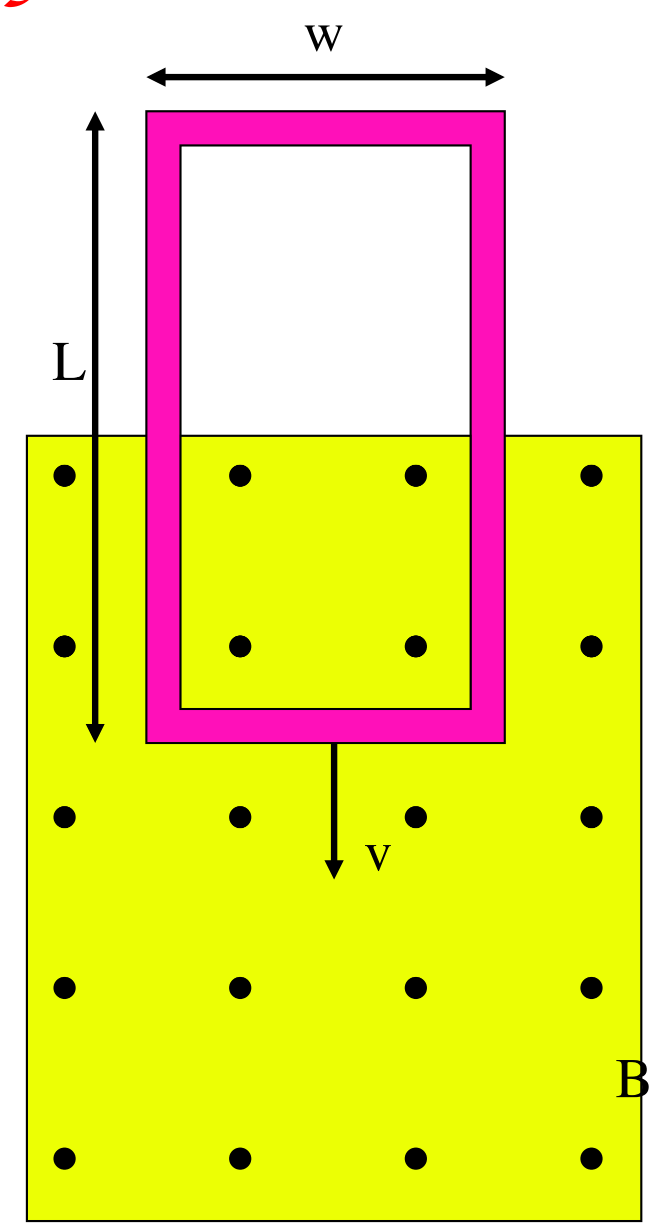
Problem 20.67 (modified)

A loop of mass m, dimension w and L and resistance R falls into a magnetic field B-field.

a.) When in the position shown, explain in detail what the direction-of-force is acting on the coil, and why you've concluded that.

The magnetic flux is increasing as the coil falls. The external B-fld is outward, so the induced B-fld must be inward. The right-hand rule maintains that this will be produced by a clockwise induced current.

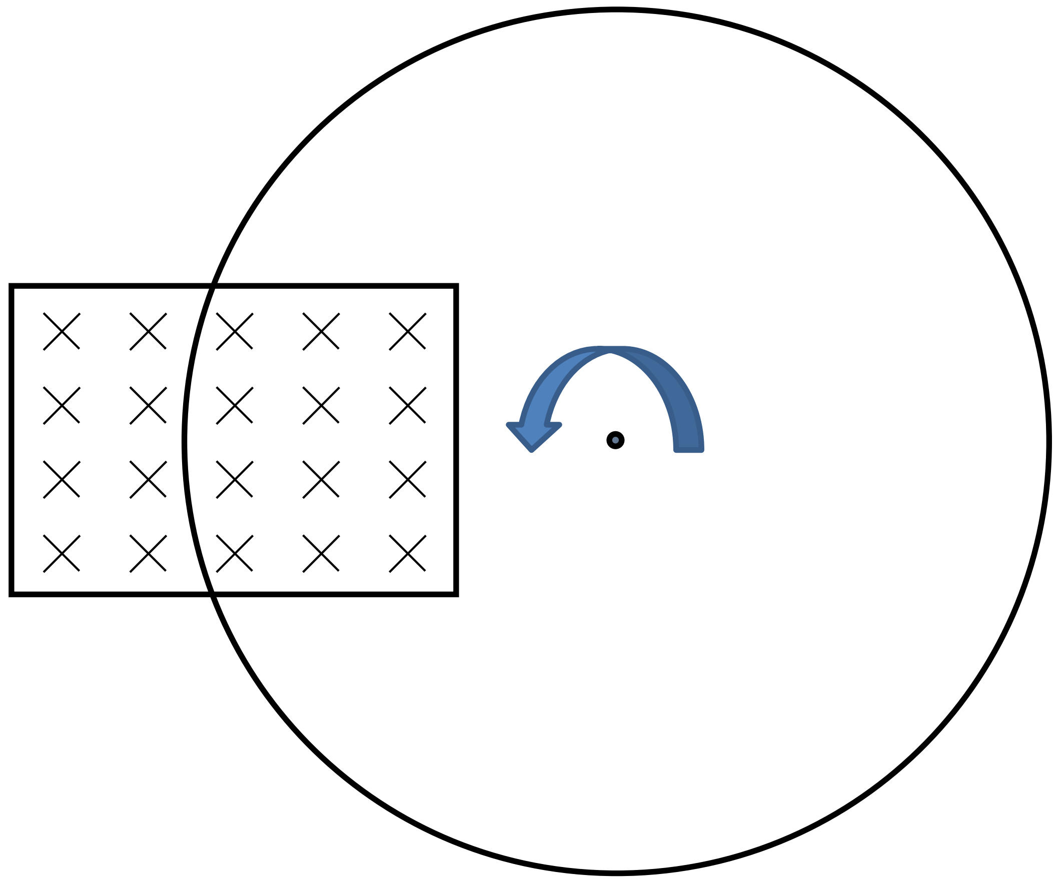
The clockwise induced current will be moving to the left in the bottom section of the loop. That means that, using iLxB, the direction of magnetic force on the wire will be upward.



Eddy current brake

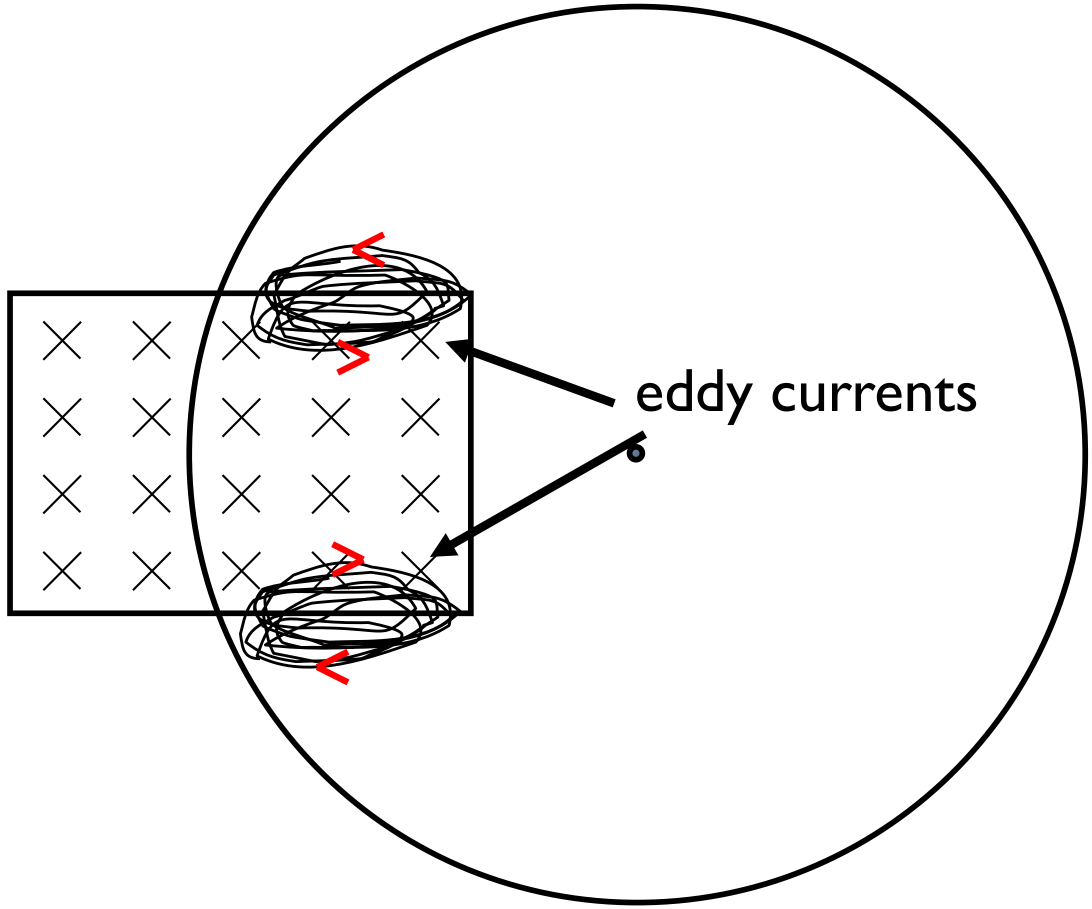
Aluminum is *not* magnetizable (it is a metal so you can get current to flow, but it doesn't have magnetic domains like iron).

So consider an aluminum disk that is made to rotate, and that has a portion of its surface continuously passing through a magnetic field directed into the page (see sketch).



Eddy current brake

At a given instant, the electrons in the area of the metal that is moving into the external B-field will experience an induced EMF which will make them swirl. The same will also happen as the metal leaves the magnetic field. This swirling of charge is called "eddy currents."

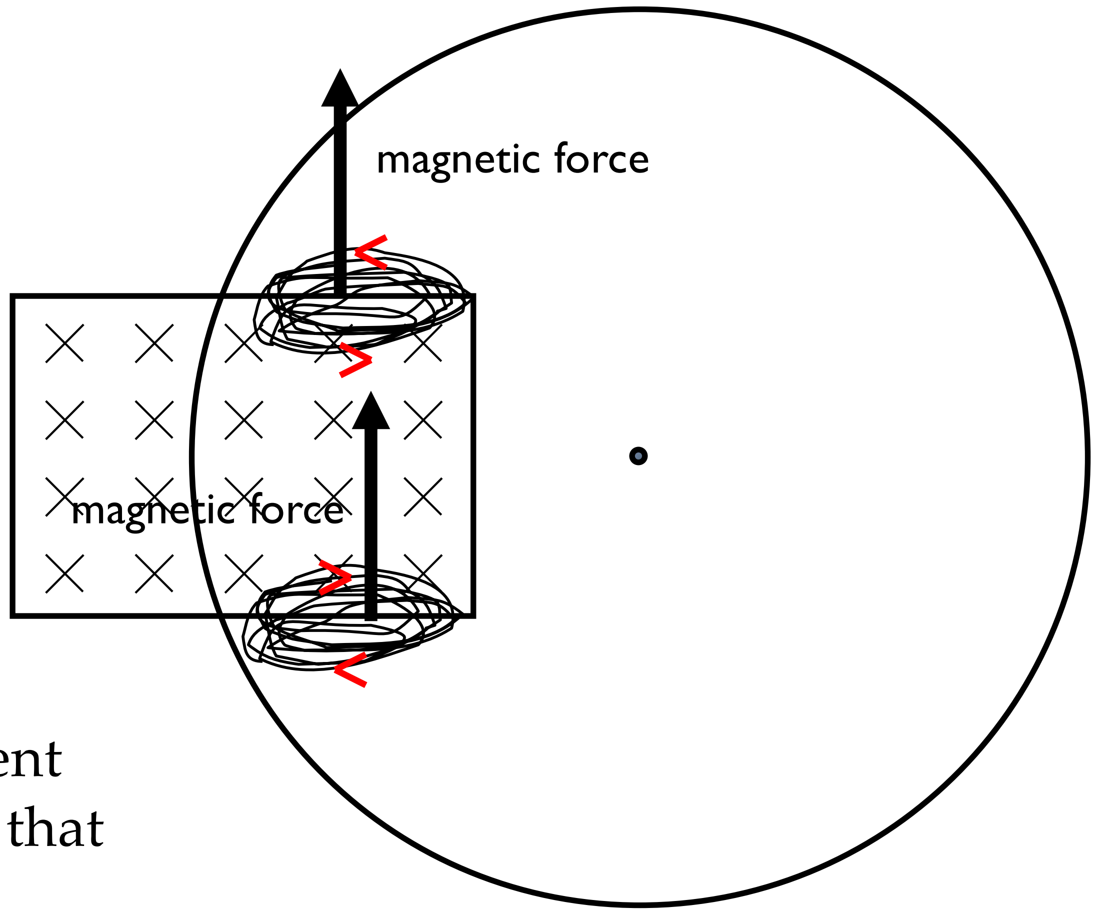


(Note that in the set-up shown, Lenz's Law suggests that the induced current (the charge swirl) in the entrance (upper) section rotates counterclockwise whereas the induced current in the exit (lower) section rotates clockwise.)

Eddy current brake

What's interesting is that when charges swirls as eddy currents, they interact with the external B-field to produce a force that fights both their entrance and exit from the field. In other words, they act like a brake.

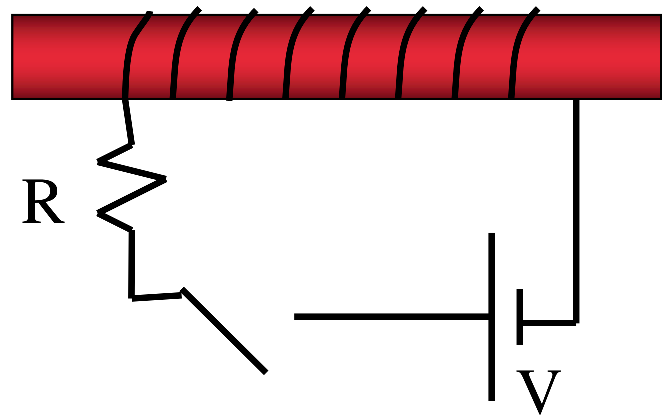
This is called an "eddy current brake." It is the mechanism that stops trains.



Inductance

Let's step back again and talk about a coil again. We've got several names for this: coil, solenoid, inductor, choke

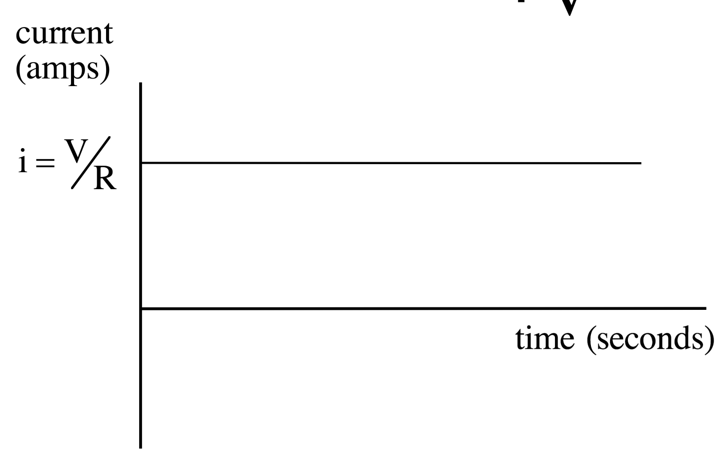
- The first two we've used a lot; the third we're about to talk about; the last is an old slang term for its use as a filter in some circuits



- Consider the situation to the right. We'll also say the coil has some resistance itself, so the total resistance of the circuit is R_{net}. At t = 0, the switch is closed.

  - If <u>nothing else was going on</u>, what would a graph of I vs t look like?

    the battery would drive current through the circuit with i = $\boldsymbol{\varepsilon}$/R and the current as a function of time would look like the function for a straight resistor circuit (see graph to right).

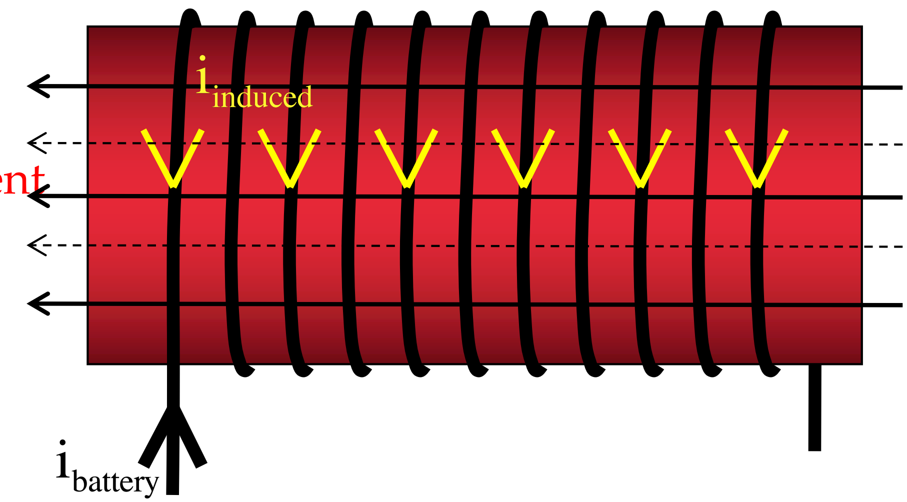


Inductance

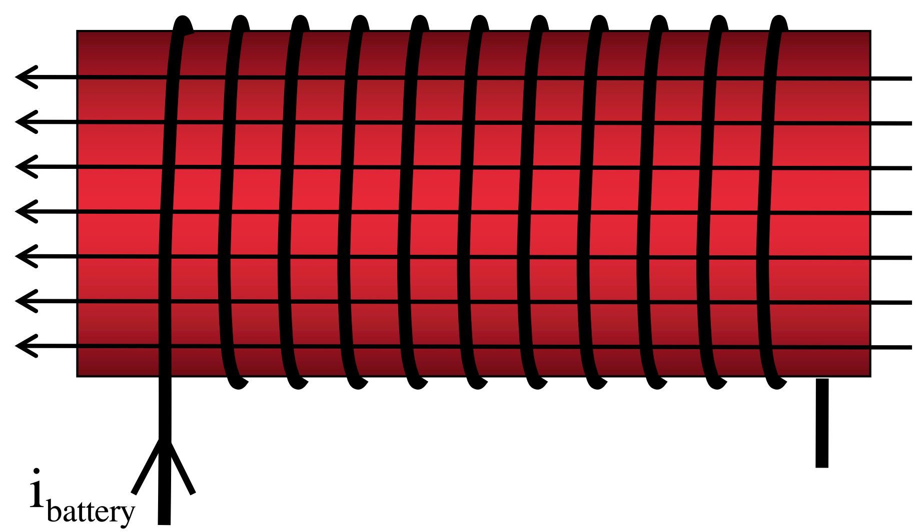
- However, there IS more going on:

As the current motivated by the battery increases, the magnetic field down the axis of the coil will increase from zero to some final value. While this is happening, the magnetic flux through the coil will increase, inducing an EMF that induces a current that is directed so as to *fight* the current change that started the chain of events. This fighting will continue until the current from the battery hits steady state.

As B-field increases, induced current fights battery current!

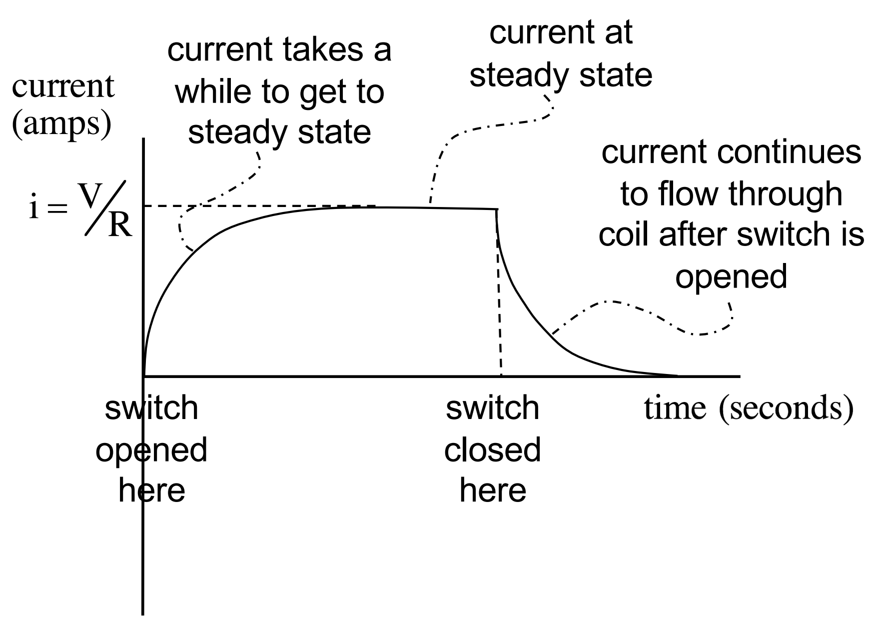


Once B-field hits its maximum, induced current goes away



Inductance

- In fact, this back-current will be so large in the beginning that the net current in the circuit will start out at *zero* (instead of immediately jumping to V/R) and will change slowly whenever a change is elicited (like when the switch is closed and when it is opened). A sketch of the situation is shown below.



Inductance

- We know from Faraday's Law that the induced EMF across the coil is

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}\left(\text{or } -N\frac{d\Phi_B}{dt} \text{ if we are using Calculus}\right),$$

but the problem with this is that it is very hard to quantitatively measure how much flux change is happening through the coil as time proceeds.

- The solution to the problem is to notice that what is *really* generating the flux change is *the change of current in time*. In other words,

$$\varepsilon_{\text{induced}} \alpha \frac{\Delta i}{\Delta t} \quad \text{or, if we are using Calculus,} \quad \varepsilon_{\text{induced}} \alpha \frac{di}{dt}.$$

Inductance

- To make this into an equality, we need a proportionality constant. Incorporating that into the mix yields:

$$\varepsilon_{induced} = -L\frac{\Delta i}{\Delta t} \quad \text{or} \quad \varepsilon_{induced} = -L\frac{di}{dt},$$

where "L" is called the *inductance* of the coil.
  - (Note that this is why coils are often called "inductors"—other monikers are "solenoid" and the ever-popular "choke.")

- L has units of Volt-seconds/Amps = ohm-seconds
  - This is also called a "Henry"

- L is related to the physical properties of the inductor (number of coils, geometry of circuit) and the greater the L, the greater resistance to changing current

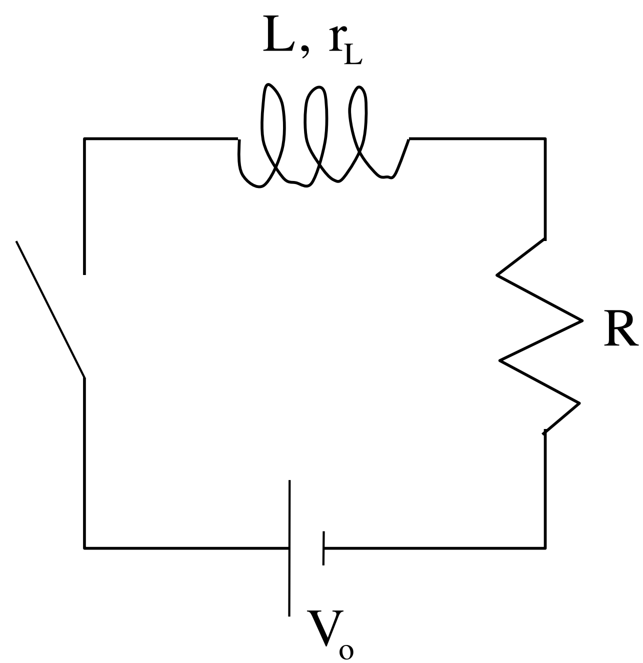
Some callback to before…

- We've seen this type of equation before…where one factor is related to another physical parameter of the circuit, related by some constant. Like…

$$V = IR$$
$$Q = CV$$
$$\varepsilon_{ind} = -L\frac{di}{dT}$$

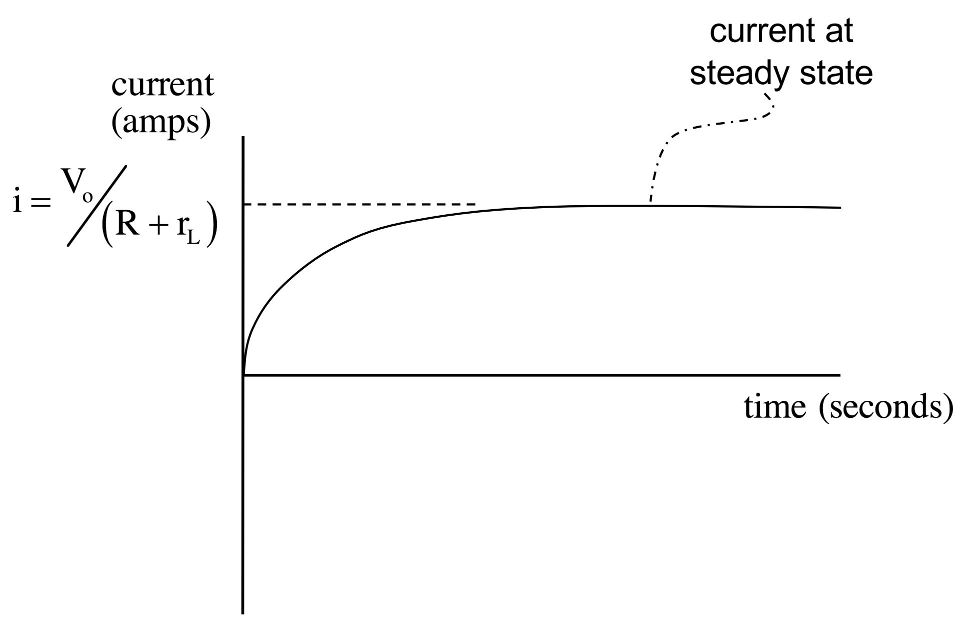
- So sketch a circuit with a resistor and inductor connected to a battery. What would the graph of current vs. time look like when the switch is closed and left for a long time?
  - When have we seen this behavior before? What did we know about it?

RL circuit

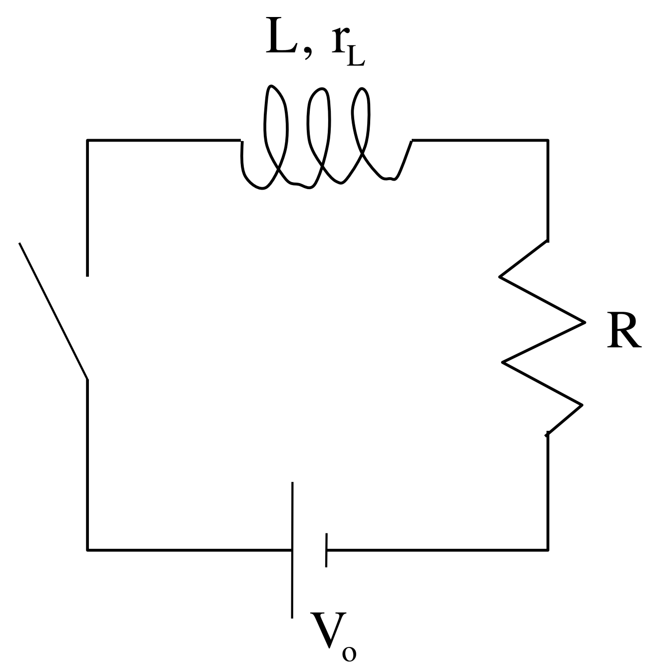


So consider the RL circuit shown to the right (note the inductor has resistor-like resistance associated with it). Assuming the switch is thrown at t=0 seconds:

As has already been pointed out, the induced EMF generated across the coil's leads as the current tries to increase to its maximum (i.e., V/R) slows the current increase in the circuit so the current versus time graph is as shown to the right:



RL circuit



It is obvious that the initial current is zero and the final current V/(R+r). It isn't, maybe, obvious what the current is between t=0 and t=infinity. To get that, we need to use Kirchoff's Laws on the RL circuit and see where it takes us. Doing so yields:

$$V_o - L\frac{d\left(i(t)\right)}{dt} - \left(i(t)\right)r_L - \left(i(t)\right)R = 0$$

$$\Rightarrow \quad \frac{d\left(i(t)\right)}{dt} + \left(\frac{R+r_L}{L}\right)\left(i(t)\right) = \frac{V_o}{L}$$

What this differential equation is asking for is a function i(t) that is such that if you take its derivative and add it to a constant times itself, you will always get the same number (V/L). As we saw with capacitors, the function that satisfies this requirement is:

$$i(t) = i_{max}\left(1 - e^{-\frac{t}{\left(L/\left(R+r_L\right)\right)}}\right).$$

RL circuit

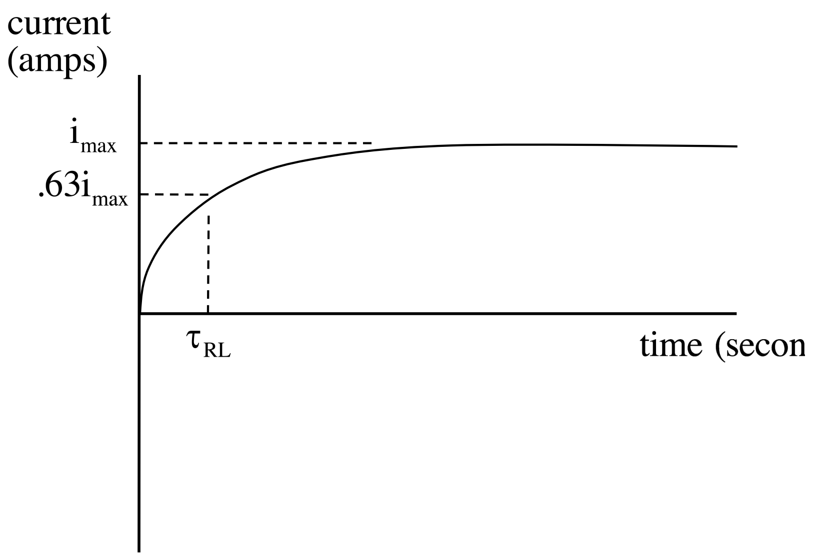
As we did with capacitors, after an amount of time t=L/(R+r) the exponent goes in the current expression goes to -1 and we get:

$$i(t) = i_{max}\left(1 - e^{-\frac{L/(R+r_L)}{(L/(R+r_L))}}\right)$$

$$= i_{max}\left(1 - e^{-1}\right)$$

$$= i_{max}\left(1 - \frac{1}{2.7}\right)$$

$$= .63 i_{max}$$



In other words, after one time-constant of

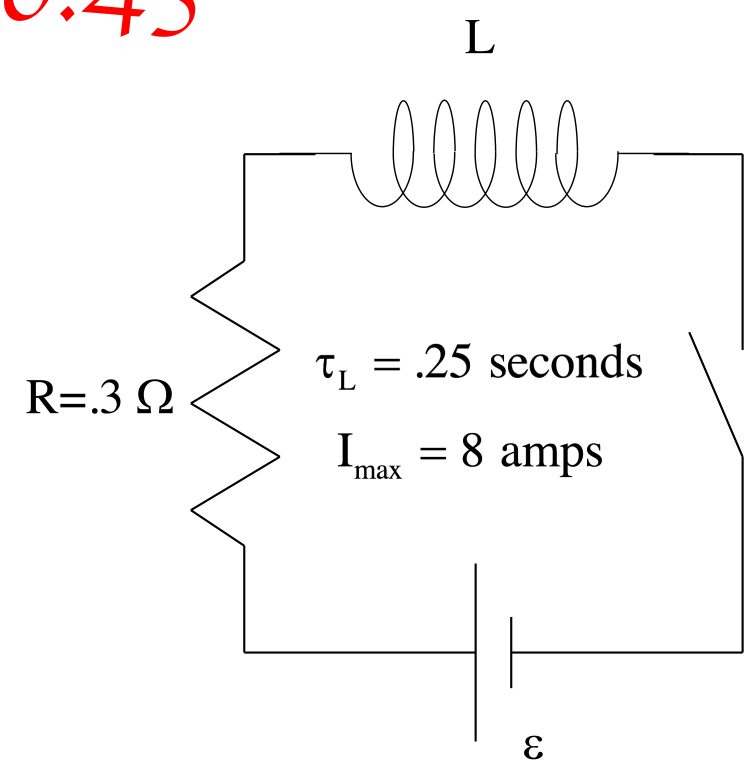
$$\tau_{RL} = \frac{L}{(R + r_L)},$$

In other words, R_{net}

the current reaches .63 of its maximum!

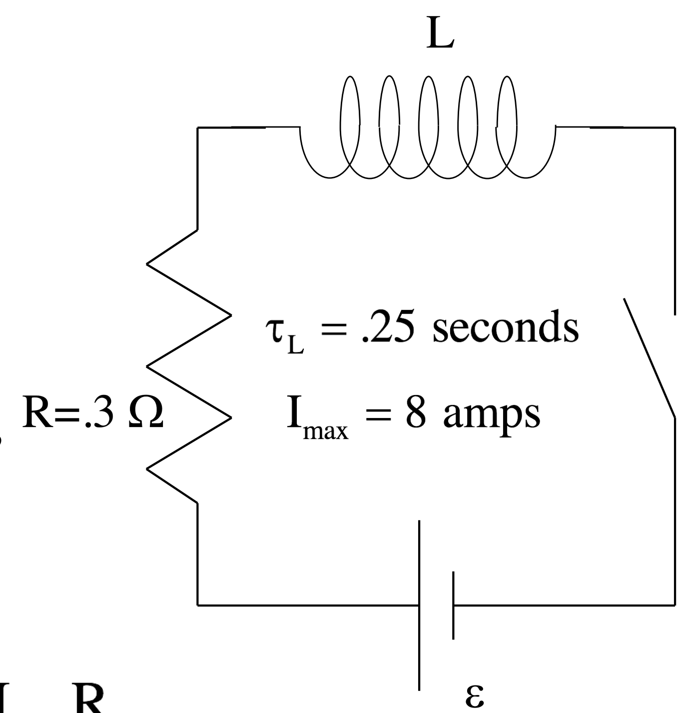
Problem 20.45

- For the circuit shown, determine:
  - A) the battery voltage
  - B) the inductance in the circuit
  - C) the current after one time constant
  - D) the voltage across the resistor after one time constant
  - E) the voltage across the inductor after one time constant



a.) the battery voltage.

In an RL circuit, there is initially no current as the inductor's back EMF will not allow current to happen. Initially, then, all the battery voltage drop occurring across the inductor. After a long time, the changing flux across the inductor's coils goes to zero, the back EMF drops to zero and all the battery voltage drop is across the resistor. That means the maximum current, which happens after a long time, will simply be:



$$\begin{aligned}\varepsilon &= I_{max}R \\ &= (8 \text{ amps})(.3\ \Omega) \\ &= 2.4 \text{ volts}\end{aligned}$$

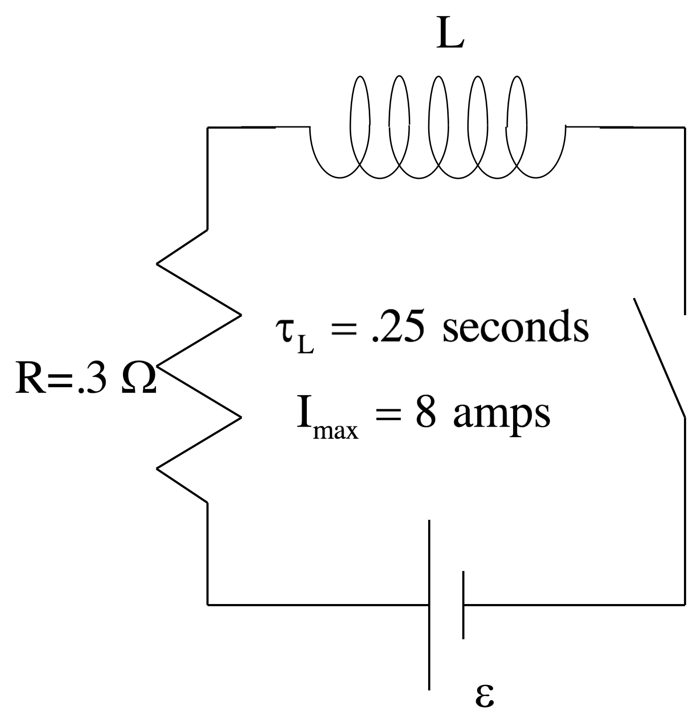
b.) the inductance in the circuit.

Using the time constant:

$$\tau_L = L/R = .25 \text{ seconds}$$

$$\Rightarrow \quad L = (.25 \text{ sec})(.3\ \Omega)$$

$$= .075 \text{ H } (= 75 \text{ mH})$$



c.) the current after one time constant.

After one time constant, the current will be at 63% of its maximum. That value is 5.04 amps.

d.) the voltage across the resistor and inductor after one time constant.

The voltage across the resistor will be ir, or:

$$\begin{aligned} V_R &= iR \\ &= (5.04\ \text{A})(.3\ \Omega) \\ &= 1.51\ \text{volts} \end{aligned}$$

e.) The voltage across the inductor is what is left over, or:

$$\begin{aligned} V_L &= \varepsilon - V_R \\ &= (2.40\ \text{volts}) - (1.51\ \text{volts}) \\ &= .89\ \text{volts} \end{aligned}$$